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New Cosmological Solutions in Massive Gravity

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We find new, simple cosmological solutions with flat, open, and closed spatial geometries, contrary to the previous wisdom that only the open model is allowed. The metric and the Stückelberg fields are given explicitly, showing nontrivial configurations of the Stückelberg in the usual Friedmann-Lemaître-Robertson-Walker coordinates. The solutions exhibit self-acceleration, while being free from ghost instabilities. Our solutions can accommodate inhomogeneous dust collapse represented by the Lemaître-Tolman-Bondi metric as well. Thus, our results can be used not only to describe homogeneous and isotropic cosmology but also to study gravitational collapse in massive gravity.

It is very intriguing to explore whether or not the graviton *can* have a mass. The first attempt to add a mass term to the gravity action was made by Fierz and Pauli [1], who considered the quadratic action for the graviton $h_{\mu\nu}$ in flat space with the mass term

$$m^2\left(h_{\mu\nu}h^{\mu\nu}-h^2\right). \tag{1}$$

The linear theory with the Fierz-Pauli mass term is ghost-free. However, the theory does not reproduce general relativity in the massless limit $m\to 0$. The extra three degrees of freedom in a massive spin 2 survive even in this limit, and therefore the prediction for light bending is away from that of general relativity, which clearly contradicts solar-system tests. This is called the vDVZ discontinuity [2].

As pointed out by Vainshtein [3], the discontinuity can in fact be cured by going beyond the linear theory. Massive gravity has a new length scale called the Vainshtein radius, below which the nonlinearities of the theory come in and the effect of the extra degrees of freedom is screened safely. The Vainshtein radius becomes larger as m gets smaller, and thereby a smooth massless limit is attained.

However, the very nonlinearities turned out to cause another trouble. Boulware and Deser argued that there appears a sixth scalar degree of freedom at nonlinear order, which has a wrong sign kinetic term, *i.e.,* the sixth mode is a ghost [4]. The ghost issue was emphasized in the effective field theory approach in Ref. [5]. The presence of the Boulware-Deser (BD) ghost has hindered us from constructing a consistent theory of massive gravity.

Recently, a theoretical breakthrough in this field has been made. Adding higher-order self-interaction terms and tuning appropriately their coefficients, de Rham and collaborators successfully eliminated the dangerous scalar mode from the theory in the decoupling limit [6, 7]. Then, Hassan and Rosen established a complete proof that the theory does not suffer from the BD ghost instability to all orders in perturbations and away from the decoupling limit [8]. Thus, there certainly exists a nonlinear theory of massive gravity that is free of the BD ghost.

In addition to the theoretical interests described above, the mystery of the accelerated expansion of the Universe [9] motivates massive gravity theories as a possible alternative to dark energy. Since the attractive force mediated by a massive graviton is Yukawa-suppressed by a factor e^{-mr}, massive gravity theories with $m\sim H_0$ (the present Hubble rate) could help if one were to avoid dark energy. Indeed, the DGP model [10], a concrete realization of massive gravity in the context of extra dimensions, admits a self-accelerating solution without the need of dark energy [11].

Then, one may wonder whether or not the massive gravity theory developed by de Rham and collaborators [6, 7] can admit flat Friedmann-Lemaître-Robertson-Walker (FLRW) cosmology. Starting from the usual FLRW metric ansatz, it has been shown that a spatially flat solution is prohibited in the above massive gravity theory [12]. Though the same argument applies to the closed model as well, this interesting fact does not hold true for the open model, and indeed the open FLRW solution has been obtained in Ref. [13]. The conclusion, however, depends upon the form of the Stückelberg fields one chooses. In other words, one can start from a nonstandard form of the cosmological metric in the unitary gauge, and then move to the usual FLRW coordinates with a nontrivial form of the Stückelberg fields, which would lead one to different conclusions.

In this *Letter*, we show that flat, closed, and open cosmological solutions can indeed be realized even in massive gravity, starting from a general Painlevé-Gullstrand (PG) metric [14, 15] in the unitary gauge. The key trick used is that the FLRW metric can be recast in a (less familiar) PG form [16]. Our new solutions also include a spherical, inhomogeneous dust collapse model described by the Lemaître-Tolman-Bondi (LTB) solution represented in the PG-type coordinates. Thus, our solutions can accommodate not only the flat cosmological

model[1] but also gravitational collapse solutions. Many other interesting solutions expressed in the PG coordinates in general relativity are also solutions to massive gravity.

The nonlinear massive gravity theory we consider is described by the action [6, 7]

$$S = \frac{M_{\rm Pl}^2}{2}\int {\rm d}^4x\sqrt{-g}\left(R+m^2\mathcal{U}\right)+S_{\rm m}, \tag{2}$$

where R is the Einstein-Hilbert term and $\mathcal{U}$ is the potential for the graviton,

$$\mathcal{U} := \mathcal{U}_2+\alpha_3\mathcal{U}_3+\alpha_4\mathcal{U}_4, \tag{3}$$

with two free parameters α_3 and α_4, in addition to the graviton mass m. Each term is defined as

$$\mathcal{U}_2 := [\mathcal{K}]^2-[\mathcal{K}^2], \tag{4}$$
$$\mathcal{U}_3 := [\mathcal{K}]^3-3[\mathcal{K}][\mathcal{K}^2]+2[\mathcal{K}^3], \tag{5}$$
$$\mathcal{U}_4 := [\mathcal{K}]^4-6[\mathcal{K}^2][\mathcal{K}]^2+8[\mathcal{K}^3][\mathcal{K}]+3[\mathcal{K}^2]^2-6[\mathcal{K}^4], \tag{6}$$

where

$$\mathcal{K}_\mu^{\ \nu} := \delta_\mu^{\ \nu}-(\sqrt{g^{-1}\Sigma})_\mu^{\ \nu}, \tag{7}$$

and rectangular brackets stand for traces. The tensor $\Sigma_{\mu\nu}$ is written in terms of four Stückelberg fields as

$$\Sigma_{\mu\nu} = \partial_\mu\phi^a\partial_\nu\phi^b\eta_{ab}, \quad \eta_{ab} = {\rm diag}(-1,1,1,1). \tag{8}$$

$S_{\rm m}$ denotes the action of matter which is minimally coupled to gravity.

The equations of motion derived from the action are of the form

$$M_{\rm Pl}^2\left(G_{\mu\nu}+m^2X_{\mu\nu}\right) = T_{\mu\nu}, \tag{9}$$

where $X_{\mu\nu}$ represents the contribution from $\mathcal{U}$ due to the graviton mass and $T_{\mu\nu}$ is the matter energy-momentum tensor. Here it should be noted that the effective energy momentum tensor $X_{\mu\nu}$ from $\mathcal{U}$ is determined only by algebraic manipulation of the inverse metric matrix g^{-1} in $\mathcal{K}_\mu^{\ \nu}$ defined by Eq. (7).

In this *Letter*, we work in a one-parameter family of the above theory, where α_3 and α_4 are given by

$$\alpha_3 = \frac{1}{3}(\alpha-1), \quad \alpha_4 = \frac{1}{12}\left(\alpha^2-\alpha+1\right). \tag{10}$$

This parameter choice (made also in Ref. [17]) enables us to find new solutions in massive gravity. In addition, this choice is useful for avoiding potential ghost instabilities suggested in Ref. [18], as would-be dangerous fluctuation modes become nondynamical from the beginning.

Our metric ansatz is taken to be the general PG form [14]:

$$\begin{aligned}{\rm d}s^2 = &-V^2(t,r){\rm d}t^2+U^2(t,r)\left({\rm d}r+\epsilon\sqrt{f(t,r)}{\rm d}t\right)^2 \\ &+W^2(t,r)r^2{\rm d}\Omega^2,\end{aligned} \tag{11}$$

where $\epsilon=\pm1$. We write the Stückelberg fields in the unitary gauge as

$$\phi^0 = t, \quad \phi^i = r\hat{n}^i, \tag{12}$$

where $\hat{n}$ is the unit radial vector, $\hat{n} = (\cos\varphi\sin\theta, \sin\varphi\sin\theta, \cos\theta)$.

In the absence of matter, the de Sitter solution has been constructed in the coordinate system of (11) [17]:

$${\rm d}s^2 = -\kappa^2{\rm d}t^2+\tilde{\alpha}^2\left({\rm d}r\pm\tilde{H}r{\rm d}t\right)^2+\tilde{\alpha}^2r^2{\rm d}\Omega^2, \tag{13}$$

where $\tilde{H} = \kappa m/\sqrt{3\alpha}$, $\tilde{\alpha} := \alpha/(\alpha+1)$, and κ is an integration constant. This solution is different from the de Sitter solution found by Koyama, Niz, and Tasinato [19, 20]. In fact, the metric (13) solves the equations of motion only in the special case where the parameters are given by Eq. (10).

We are going to generalize the work of Ref. [17] to accommodate a wider class of dynamical solutions including cosmological ones. In light of the de Sitter solution (13), we concentrate on the case satisfying

$$W(t,r) = \tilde{\alpha} := \frac{\alpha}{\alpha+1}. \tag{14}$$

The key observation here is that *for any metric of the form (11) with (14) and for the Stückelberg (12), the tensor $X_{\mu\nu}$ reduces to the effective cosmological constant term:*

$$X_{\mu\nu} = \frac{1}{\alpha}g_{\mu\nu}. \tag{15}$$

This happens only for the special parameter choice (10). Consequently, *any PG-type solution in general relativity (with a cosmological constant) is also a solution to massive gravity.* Equation (15) implies that the identity $M_{\rm Pl}^2m^2\nabla_\mu X^{\mu\nu} = \nabla_\mu T^{\mu\nu}-M_{\rm Pl}^2\nabla_\mu G^{\mu\nu} = 0$ is automatically satisfied. Our finding thus extends the observations made in Refs. [17, 21] to the general PG metric.

Let us demonstrate how cosmological solutions are obtained using the above fact. The FLRW metric can be rewritten in a general PG form as [15, 22]

$$\begin{aligned}{\rm d}s^2 = &-\kappa^2{\rm d}t^2+\frac{\tilde{\alpha}^2}{1-K\tilde{\alpha}^2r^2/a^2(t)}\left({\rm d}r-\frac{\dot{a}}{a}r{\rm d}t\right)^2 \\ &+\tilde{\alpha}^2r^2{\rm d}\Omega^2,\end{aligned} \tag{16}$$

[1] Though inflation does not necessarily predict an exactly flat universe, such a solution is useful for describing our Universe and investigating the cosmological perturbations.

where $K = 0, \pm 1$, a dot denotes differentiation with respect to t, and we have taken $\epsilon = -1$ for expanding flow. In general relativity, one is free to rescale the time coordinate to remove the constant κ. However, this is not the case in massive gravity because such a rescaling will change the tensor $\Sigma_{\mu\nu}$, and then $\Sigma_{\mu\nu}$ will depend on κ. For this reason, we do not set $\kappa = 1$ but rather leave κ in the metric as an integration constant characterizing the solution.

We include a perfect fluid whose energy momentum tensor is given by

$$T_{\mu\nu} = (\rho + p)\, u_\mu u_\nu + p g_{\mu\nu}, \tag{17}$$

where $u_\mu = (-V, 0, 0, 0)$. The energy density and pressure may depend on both t and r in general, but in the present cosmological setting they are supposed to depend only on t: $\rho = \rho(t), p = p(t)$.

The equations of motion (9) now read

$$\frac{3}{\kappa^2}\tilde{H}^2 = \frac{\rho}{M_{\rm Pl}^2} + \frac{m^2}{\alpha} - \frac{3K}{a^2}, \tag{18}$$

$$-\frac{1}{\kappa^2}\left(3\tilde{H}^2 + 2\dot{\tilde{H}}\right) = \frac{p}{M_{\rm Pl}^2} - \frac{m^2}{\alpha} + \frac{K}{a^2}, \tag{19}$$

where $\tilde{H}(t) := {\rm d}\ln a/{\rm d}t$. From the energy conservation equation, $u_\mu \nabla_\nu T^{\mu\nu} = 0$, we obtain

$$\dot{\rho} + 3\tilde{H}(\rho + p) = 0. \tag{20}$$

Rescaling the time coordinate as

$$t \to \tau = \kappa t, \tag{21}$$

and using $H := {\rm d}\ln a/{\rm d}\tau$ instead of $\tilde{H}$, one can apparently remove the constant κ from the above equations. Thus, the standard cosmological equations with the effective cosmological constant

$$\Lambda_{\rm eff} = \frac{m^2}{\alpha} \tag{22}$$

are reproduced. It should be emphasized that in the present case *spatially flat, open and closed models are possible*. This is in sharp contrast to the findings in Refs. [12, 13]. It is straightforward to include a "bare" cosmological constant Λ by shifting $\Lambda_{\rm eff} \to \Lambda + m^2/\alpha$.

The metric (16) may be expressed in a more familiar "cosmological" form. This is done by using the new radial coordinate defined as

$$r \to \varrho = \frac{\tilde{\alpha} r}{a(\tau)}. \tag{23}$$

In terms of ϱ, the metric is indeed of the FLRW form:

$${\rm d}s^2 = -{\rm d}\tau^2 + a^2\left(\frac{{\rm d}\varrho^2}{1 - K\varrho^2} + \varrho^2 {\rm d}\Omega^2\right). \tag{24}$$

However, this coordinate transformation brings the Stückelberg scalar fields to

$$\phi^0 = \frac{\tau}{\kappa}, \quad \phi^i = \frac{a(\tau)\varrho}{\tilde{\alpha}}\hat{n}^i, \tag{25}$$

so that

$$\Sigma_{\mu\nu}{\rm d}x^\mu {\rm d}x^\nu = -\left(\frac{1}{\kappa^2} - \frac{a^2 H^2 \varrho^2}{\tilde{\alpha}^2}\right){\rm d}\tau^2 + 2\frac{a^2 H \varrho}{\tilde{\alpha}^2}{\rm d}\tau {\rm d}\varrho + \frac{a^2}{\tilde{\alpha}^2}\left({\rm d}\varrho^2 + \varrho^2 {\rm d}\Omega^2\right). \tag{26}$$

Thus, we see that, though the geometry described by the metric (24) is spatially homogeneous and isotropic, the tensor $\Sigma_{\mu\nu}$ does not respect the same symmetry. After the coordinate transformation (21) and (23), $\Sigma_{\mu\nu}$ carries the information about the constants κ and $\tilde{\alpha}$.

Introducing the Stückelberg fields gives rise to a new invariant $I^{ab} = g^{\mu\nu}\partial_\mu \phi^a \partial_\nu \phi^b$. It is easy to check that the solution we have obtained shows no singularity in I^{ab}. This owes to the fact that the metric of the PG form has no coordinate singularity on its horizon.

It would be an important next step to study perturbations around our cosmological background. The analysis of cosmological perturbations will be nontrivial because the reference metric does not respect the same symmetry as the FLRW one. However, fluctuations around the de Sitter background have been investigated in the decoupling limit in Ref. [17], and it has been found that the kinetic terms for the helicity-0 and helicity-± 1 modes vanish identically. This is the consequence of the special parameter choice (10). In particular, as mentioned above, this implies that one can avoid potential ghost instabilities suggested in Ref. [18].

Finally, we mention the description of gravitational collapse in massive gravity in terms of the PG-type solutions. Our cosmological metric in the PG coordinates can also be utilized to describe collapsing matter in spherically symmetric spacetime (with $\epsilon = +1$ to adapt the contracting setup). Indeed, the metric of the PG form has been used to analyze the spherical contraction model of a star with uniformly distributed dust [22], and with a perfect fluid [15], while in the present case the Schwarzschild-de Sitter solution found in Ref. [17] is to be used to describe the exterior solution. In addition, we explicitly show that the LTB metric representing general inhomogeneous collapsing dust is also included in the class of our PG-type solutions.

In general relativity, the metric representing spherically symmetric spacetime can be expressed in a general PG form without loss of generality as

$${\rm d}s^2 = -N^2 {\rm d}t^2 + \frac{\tilde{\alpha}^2}{1 + 2E}(N_r {\rm d}t + {\rm d}r)^2 + \tilde{\alpha}^2 r^2 {\rm d}\Omega^2, \tag{27}$$

where $N(t, r) > 0$ is the lapse function, $N_r(t, r)$ is the radial component of the shift vector, and $E(t, r) > -1$.

The energy momentum tensor $T_{\mu\nu}$ is given by Eq. (17) while ρ and p are no longer homogeneous, and a "bare" cosmological constant Λ may also be included. The metric (27) can be used to analyze spherical collapse of a perfect fluid [24].

From the equation of motion, we see

$$E = \frac{1}{2}\left(\frac{\tilde{\alpha}N_r}{N}\right)^2 - \frac{M}{\tilde{\alpha}r}, \tag{28}$$

where we defined an enclosed mass

$$M(r,t) := 4\pi \int^r \left(\rho + M_{\rm Pl}^2\Lambda\right) {r'}^2 {\rm d}r'. \tag{29}$$

Here, E represents energy, which is conserved for dust, as shown later.

The analytic solution for general inhomogeneous dust collapse is most commonly expressed in the LTB coordinates. We change the coordinates from (t,r,θ,ϕ) to (T,R,θ,ϕ) such that $t = t(T) = T$ and $r = r(T,R) = \bar{r}(T,R)/\tilde{\alpha}$ with

$$\left(\frac{\partial \bar{r}}{\partial T}\right) = -\tilde{\alpha}N_r = -N\sqrt{\frac{2M}{\bar{r}} + 2E}, \tag{30}$$

where we have taken only the positive root of Eq. (28) for a collapsing fluid. Now, N, M, and E are all functions of T and R, and the metric is of the LTB form:

$${\rm d}s^2 = -N^2{\rm d}T^2 + \frac{1}{1+2E}\left(\frac{\partial \bar{r}}{\partial R}\right)^2 {\rm d}R^2 + \bar{r}^2{\rm d}\Omega^2. \tag{31}$$

In the case of dust in which one can eliminate the pressure gradient from the relevant equations, N is independent of R and hence is allowed to be synchronous, $N = 1$. However, for the application to massive gravity, it would be better to leave N in the metric as an additional arbitrary function of T. Moreover, it turns out that E is independent of T, and then the remaining evolution equations have three types of solutions depending on the sign of $E(R)$. This is the well known LTB solution that can be expressed in the PG form (27) as well.

On the other hand, in massive gravity with the parameter choice (10), one immediately sees that the LTB solution in the PG form (27) with a cosmological constant $\Lambda + m^2/\alpha$ is the solution to (9) in the unitary gauge. This is because, as mentioned above, given metric functions of the generalized PG form, the effective energy momentum tensor $X_{\mu\nu}$ of $\mathcal{U}$ reduces to the cosmological term, $(1/\alpha)g_{\mu\nu}$, irrespective of their coordinate dependence. The coordinate transformation from PG to LTB with the rescaling $T \to T/N$ brings the Stückelberg scalar fields to

$$\phi^0 = \int \frac{{\rm d}T}{N}, \quad \phi^i = \frac{\bar{r}}{\tilde{\alpha}}\hat{n}^i, \tag{32}$$

so that

$$\Sigma_{\mu\nu}{\rm d}x^\mu {\rm d}x^\nu = -\frac{{\rm d}T^2}{N^2} + \left(\frac{\partial \bar{r}}{\partial T}\frac{{\rm d}T}{\tilde{\alpha}} + \frac{\partial \bar{r}}{\partial R}\frac{{\rm d}R}{\tilde{\alpha}}\right)^2 + \left(\frac{\bar{r}}{\tilde{\alpha}}\right)^2 {\rm d}\Omega^2. \tag{33}$$

Note that also in this case the invariant I^{ab} does not diverge on the horizon by virtue of the PG coordinate system.

In summary, we have found new cosmological solutions in massive gravity with flat, open, and closed spatial geometries. Our solutions can also describe inhomogeneous gravitational collapse of dust represented by the LTB metric. The key was that general PG-type metric gives rise to an effective energy momentum tensor of a cosmological constant m^2/α for the special choice of the parameters. This is essential to hunt for analytical solutions in massive gravity from the seed solutions in general relativity. Thus, our solutions can be used not only for homogeneous and isotropic cosmology with arbitrary spatial curvature, but also for the spherical collapse model of the formation of cosmic structure such as stars and galaxies.

In this *Letter*, the special choice of the parameters of the theory enabled us to have the desirable structure $X_{\mu\nu} \propto g_{\mu\nu}$. Consequently, the conservation law $\nabla_\mu X^{\mu\nu} = 0$ is satisfied automatically. The presence of the conservation law suggests the presence of some accidental symmetry in the PG-type metric. It would be thus interesting to understand more deeply the nature of the metrics giving the effective cosmological term. As a concrete example, we will report the behavior of perturbations on our cosmological background in a separate publication [25].

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Note added While this paper was being completed, Ref. [26] appeared, in which a similar situation is discussed where an effective cosmological term $X_{\mu\nu} \propto g_{\mu\nu}$ arises. The authors of Ref. [26] derived a constraint that is to be imposed on the metric and the Stückelberg functions, while we have given explicit solutions for the metric and the Stückelberg fields in a simple manner.

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